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DISSERTATIO ACADEMICA

THEORIAM ÆQUATIONUM FUNCTIONALIUM DUARUM VARIABILIUM EJUSQUE IN DOCTRINA SERIERUM USUM EXHIBENS;

QUAM

CONSENSU AMPLISS. FACULTATIS PHILOSOPH.

AD IMPERIALEM ACAD. ABOËNSEM,

PRÆSIDE

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PRO GRADU PHILOSOPHICO

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P. I.

ABOÆ, Typis FRENCKELLIANIS.

§. I.

Doctrinam æquationum *Functionalium* dictarum nomine adeo in Analysi late patere, ut nulla fere ad eam reduci non queant ejus problemata, sive resolutionem æquationum, sive doctrinam serierum, sive Calculum etiam Integralem ista respiciant, cuicumque præsentis Mathematum status gnaro notum satis est. Licet autem sic nostris demum temporibus eo perventum sit, ut ad problema universale omnes fere Analyseos Mathematicæ difficultates revocare possimus, hoc tamen ipso ad easdem revera extricandas parum omnino nos profecisse fatendum est, cum, exceptione facta casuum quorundam valde particularium, inexsuperabilibus fere adhuc prematur difficultatibus tractatio memorati generis æquationum. Universalissimas hoc in genere disquisitiones Mathematicis summi nominis *Eulero*, *Lagrangio* atque *Laplacio* debemus; ut taceamus autem hasce casum tantum simplicissimum, duarum scilicet variabilium, præcipue respicere, vel pro eo etiam ipso arctissimis circumscripta sunt limitibus quæ hoc in argumento eruere valuerunt laudati Auctores resultata. Hæc vero licet ita se habeant, attentione tamen dignissima eadem nobis videntur resultata, ut scilicet non tantum per se admodum pulchra, sed etiam, ut alias eorum applicationes silentio prætereamus, in theoria præsertim serierum usus egregii, cum ad istam scilicet doctrinam sub concinna commodaque exhibendam forma apprime sint idonea. Quæ quidem perpendentes inutilem nos non sumturos operam credidimus, si uberiorem aliquanto conciliare iisdem lucem conaremur, proposita nimirum hacce materie sub forma, ut nobis videtur, concinniori applicationibusque aptiori ac ab Auctoribus tractari vulgo solet, hincque præsens, quod benignæ lectorum censuræ jam offerimus, opusculum ortum est.

§. II.

Æquatio quæcumque functionalis duarum variabilium sub sequenti forma exhiberi potest

$$F^{\prime} \left(u,\ y_{\varphi u},\ y_{\varphi_1 u},\ y_{\varphi_2 u},\ \ldots\ y_{\varphi_n u}\right) = 0 \ldots \mathrm{I}),$$

ubi

ubi F' φ, φ_1, φ_2, .. φ_n functiones sunt formæ datæ, ipsa autem y functio illa incognita, cujus determinationem *resolutionem* æquationis 1) in sequentibus dicemus, hacce igitur voce in latiori ac in Analysi usurpari vulgo solet sensu assumta. Absolvitur igitur resolutio æquationis 1) per investigationem functionis ejusmodi formæ quantitatis cujuscumque z, ut, positis in eadem pro z ipsis φu, $\varphi_1 u$, $\varphi_2 u$, .. $\varphi_n u$, valoribusque ejusdem hinc prodeuntibus pro $y_{\varphi u}$, $y_{\varphi_1 u}$, $y_{\varphi_2 u}$, .. $y_{\varphi_n u}$ in ipsa 1) substitutis, identica fiat haec æquatio omnibus pro valoribus quantitatis u, quæ indeterminata igitur omnino manere ponenda est. Determinato sic in genere problemate, in quo solvendo jam occupabimur, Analytico, quo natura ejus plenius adhuc illustretur, exempla sequentia particularia subjungere juvabit.

Pro æquatione e. g. functionali

$$y_{u+1} \cdot y_{u-1} - u^2 + 1 = 0$$

habebitur

$$y_z = z,$$

quia scilicet

$$(u+1)(u-1) - u^2 + 1 = 0$$

 æquatio

æquatio habetur pro omnibus ipsius u valoribus identica.

Pro æquatione

$$y_{\sqrt{u}} + 2u^2 . y_{\sqrt[3]{u}} + 3u^3 . y_{\sqrt[4]{u}} - 6u^6 = o$$

fiet

$$y_z = z^{12},$$

cum sit

$$(\sqrt{u})^{12} + 2u^2 (\sqrt[3]{u})^{12} + 3u^3 (\sqrt[4]{u})^{12} - 6u^6 = o$$

sine ulla ipsius u determinatione identica.

In æquatione

$$y_{u^2} - y_{u+2} + \frac{u-2}{u^2-1} = o$$

erit

$$y_z = \frac{1}{z-1}$$

ob

$$\frac{1}{u^2-1} - \frac{1}{u+2-1} + \frac{u-2}{u^2-1} = o$$

in genere identicam.

In

In æquatione

$$y_{u+1} - y_u^2 + 2 = 0$$

habebitur

$$y_z = a^{2^{z-1}} + \frac{1}{a^{2^{z-1}}}$$

designante a quantitate quacumque a z non pendente, quia scilicet

$$a^{2^{u+1-1}} + \frac{1}{a^{2^{u+1-1}}} - \left(a^{2^{u-1}} + \frac{1}{a^{2^{u-1}}}\right)^2 + 2 = 0$$

in genere identica.

Æquationi denique

$$y_{u+2} - 2y_{u+1} + 2y_u - u = 0$$

respondebit

$$y_z = 2^{\frac{z}{2}} \left(a \,.\, \mathrm{Sin}\, \frac{\pi z}{4} + b \,.\, \mathrm{Cos}\, \frac{\pi z}{4}\right) + z,$$

ubi $\pi = 3,14159$ &c. nec non a et b quantitates quæcumque z non continentes, cum sit scilicet

$$\left.\begin{aligned}&2^{\frac{u+2}{2}}\left(a\,.\,\mathrm{Sin}\,\frac{\pi(u+2)}{4}+b\,.\,\mathrm{Cos}\,\frac{\pi(u+2)}{4}\right)+u+2\\-2\,.\,&2^{\frac{u+1}{2}}\left(a\,.\,\mathrm{Sin}\,\frac{\pi(u+1)}{4}+b\,.\,\mathrm{Cos}\,\frac{\pi(u+1)}{4}\right)-2\,(u+1)\\&+2\,.\,2^{\frac{u}{2}}\left(a\,.\,\mathrm{Sin}\,\frac{\pi u}{4}+b\,.\,\mathrm{Cos}\,\frac{\pi u}{4}\right)+2u-u\end{aligned}\right\}=o,$$

id est

$$\left.\begin{aligned}&2\,.\,2^{\frac{u}{2}}\left(a\,.\,\mathrm{Cos}\,\frac{\pi u}{4}-b\,.\,\mathrm{Sin}\,\frac{\pi u}{4}\right)+u+2\\&-2\,.\,2^{\frac{u}{2}}\left(a\,.\,\mathrm{Sin}\,\frac{\pi u}{4}+a\,.\,\mathrm{Cos}\,\frac{\pi u}{4}+b\,.\,\mathrm{Cos}\,\frac{\pi u}{4}\right.\\&\qquad\qquad\left.-\,b\,.\,\mathrm{Sin}\,\frac{\pi u}{4}\right)-2u-2\\&+2\,.\,2^{\frac{u}{2}}\left(a\,.\,\mathrm{Sin}\,\frac{\pi u}{4}+b\,.\,\mathrm{Cos}\,\frac{\pi u}{4}\right)+2u-u\end{aligned}\right\}=o,$$

æquatio omnino identica.

Sicque porro.

§. III.

Cum tantis prematur difficultatibus resolutio æquationis universalis 1), ut in genere exhiberi minime queat, quinam illius considerandi heic sint casus particulares ante omnia definire convenit. Observandum hunc in finem primum est, æquationem

nem ipsam 1) ad faciem aliquanto simpliciorem revocari posse, si variabilis u in aliam x mutetur, posito

$$\varphi u = x.$$

Valore ipsius u hinc eruendo, prodeat

$$u = \varphi' x \quad *);$$

eoque in ipsa 1) substituto, mutabitur ea in

$$F'(\varphi' x,\ y_x,\ y_{\varphi_1 \varphi' x},\ y_{\varphi_2 \varphi' x},\ \ldots\ y_{\varphi_n \varphi' x}) = o,$$

hoc est

$$F''(x,\ y_x,\ y_{f_1 x},\ y_{f_2 x},\ \ldots\ y_{f_n x}) = o\ \ldots\ .\ 2),$$

ubi F'', f_1, f_2, .. f_n functiones sunt datæ formæ.

Æquationis istius 2), ad quam reductam igitur videmus allatam supra 1), universalis maxime quem in genere tractare valuerunt Mathematici

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*) Contineri perspicuum est determinationem istam ipsius u in resolutione æquationis generalis 1), induente scilicet ea hoc in casu formam simplicissimam

$$y_{\varphi u} - u = o:$$

considerationi vero hujusce æquationis nostræ 1) casus particularis, qui resolutione scilicet æquationum *vulgari* absolvitur, in præsenti opella non immorabimur.

tici casus, quemque igitur tantum in sequentibus consideraturos nos esse jam ab initio monemus, is quidem est, quo ea functiones inter $f_1, f_2, \ldots . f_n$ particularis obtinet relatio, ut sit

$$f_2\, x = f_1\, f_1\, x$$
$$f_3\, x = f_1\, f_2\, x = f_1\, f_1\, f_1\, x$$
$$f_4\, x = f_1\, f_3\, x = f_1\, f_1\, f_1\, f_1\, x$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$f_n\, x = f_1\, f_{n-1}\, x = f_1\, f_1\, f_1 \,.\,.\, f_1\, x.$$

Sic v. gr. casus simplicissimus, quo

$$f_1\, x = 2x, \quad f_2\, x = f_n\, x = 3x,$$

utcumque sit ille particularis, sequentibus tamen disquisitionibus non comprehenditur.

Brevitatis gratia ponendo

$$f_1\, f_1\, x = f_1^{\,2}\, x$$
$$f_1\, f_1\, f_1\, x = f_1^{\,3}\, x$$
$$f_1\, f_1\, f_1\, f_1\, x = f_1^{\,4}\, x$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$f_1\, f_1\, f_1 \,.\,.\, f_1\, x = f_1^{\,n}\, x,$$

signum-